AP® Calculus BC

Course Description:

The prerequisite for this course is Pre-Calculus with Trigonometry (preferably Honors) including a clear understanding of the unit circle. Students should be ready for the challenge of learning AP® Calculus AB (Calculus I) and AP® Calculus BC (Calculus II) as a two-semester course. Taking the AP® Calculus BC exam for university credit is not a requirement of this course. Since we cover both Calculus I and Calculus II material over a single school year, the pace is brisk. This is a challenging course that uses a college level textbook, written by James Stewart, which includes all the necessary concepts to prepare for a rigorous college calculus class and/or the AP exam.

Calculus is the study of change and motion. Our problems "move" makes the study of Calculus exciting and engaging. In Calculus I, we discuss limits, derivatives, applications of the derivative, integrals and their applications, and an introduction to differential equations. Continuing with Calculus II, we explore further L'Hospital's Rule and additional integration techniques. Then we will move our discussions to sequences and series including Taylor and Maclaurin Series and learn how to represent functions as power series. Euler's Method, logistic growth models, parametric curves, vectors, polar curves will also be discussed.

Assignments and Assessments:

<u>Each unit will be completed following this process:</u>

1. Students will attend live and interactive classes or view recorded (At My Pace - AMP) instruction video and participate by answering questions in the live setting or by pausing the AMP instruction video to solve a given problem. Students will have immediate feedback on their current level of comprehension, thus reinforcing learning and providing the opportunity to improve their skills within each lesson.
2. Students will complete one homework assignment before attending or viewing the next instruction session. Students are provided with answers to homework assignments and are expected to self-check their answers. Classes begin with time for students to ask the instructor about homework questions they were not able to complete correctly. Students using AMP recorded instruction can set up time with an instructor, up to 5 hours per course, for individual tutoring.
3. After approximately every two assignments, students complete a quiz (using paper and pencil) that is proctored by an adult and returned to the instructor for grading and feedback. Instructors look at both work and answers to ensure that students are using sound mathematical processes to demonstrate mastery. Students complete tests over larger sections of material in the same manner. The instructor creates an online personal grading notebook for each student where the student's work, the instructor's feedback, and the grade can be viewed. Access to the notebook is granted to the student and parents, as well as any education advisors.
4. At the end of each semester, students complete a cumulative final exam.
5. Course grades are assigned based on a weighted average of 40% quiz / 60% test. The final exam is weighted the same as the other tests.

Unit 1: Limits and Continuity

Description:

Unit 1 introduces the concepts of limits and continuity. Students will learn proper mathematical notations and procedures when determining the value of limits using graphs, tables and algebraic principles and manipulation. Continuity and discontinuity will be explored and as well as how certain limits define vertical and horizontal asymptotes.

Skills to be demonstrated:

- Define Instantaneous rates of change.
- Define limits and use limit notation.
- Estimate limit values using graphs and tables.
- Determine limits using algebraic properties and manipulation.
- Select appropriate procedures for determining limits including the Squeeze Theorem.
- Connect multiple representations of limits.
- Discern types of discontinuities.
- Define continuity at a point and over an interval.
- Remove discontinuities.
- Connect infinite limits to vertical asymptotes and limits at infinity to horizontal asymptotes.
- Work with the Intermediate Value Theorem.

Unit 2: Differentiation: Definition and Fundamental Properties

Description:

Unit 2 introduces the concepts of the derivative. The definition of the derivative will be discussed, and students will gain both a visual and algebraic understanding of the derivative. Techniques of finding derivatives will be explored and practiced. Students will understand the difference between instantaneous and average rates of change.

Skills to be demonstrated:

- Define average and instantaneous rates of change at a point.
- Define the derivative of a function and use derivative notation.
- Estimate derivatives of a function at a point.
- Connect differentiability and continuity and discern when a derivatives exists and does not exist.
- Apply the Power Rule.
- Find derivatives using differentiation rules (constant, sum, difference, constant multiple).
- Find derivatives for trigonometric, exponential, logarithmic functions.
- Find derivatives using the Product Rule and Quotient Rule.

Unit 3: Differentiation: Composite, Implicit and Inverse Functions

Description:

Unit 3 continues with the concept of the derivative. Chain Rule and implicit differentiation will be introduced along with finding derivatives of inverse functions. Higher order derivatives will be presented and their applications.

Skills to be demonstrated:

- Find derivatives using the Chain Rule.
- Find derivatives using implicit differentiation.
- Differentiate inverse functions including inverse trigonometric functions.
- Select procedures for calculating derivatives.
- Calculate higher order derivatives.

Unit 4: Contextual Applications of Differentiation

Description:

Unit 4 introduces the real-world meaning of the derivative. Students will be able to write a statement to explain what a derivative and rates of change are using various contexts. Straight-line motion will be explored. Related rates word problems will be practiced using various real-world contexts. Local linearity and linearization will be taught, and this unit ends with applying L'Hospital's Rule to indeterminate limits.

Skills to be demonstrated:

- Interpret the meaning of the derivative in context.
- Explore straight-line motion (position, velocity and acceleration).
- Interpret rates of change in applied contexts other than motion.
- Solve related rates word problems.
- Approximate values of a function using local linearity and linearization.
- Use L'Hospital's Rule for determining limits of indeterminate forms.
- Discern between various indeterminant limits.
- Select appropriate techniques to evaluate limits using L'Hospital's Rule.

Unit 5: Analytical Applications of Differentiation

Description:

Unit 5 introduces the Mean Value Theorem and Extreme Value Theorem. Students will discern between global and local extrema and critical points. Graphing functions will be completed by using the first and second derivatives showing increasing/decreasing and concavity. Both the first and second derivative test will be taught to determine extrema. This unit concludes with solving real-world optimization problems.

Skills to be demonstrated:

- Use the Mean Value Theorem.
- Apply the Extreme Value Theorem.
- Discern between global and local extrema and critical points.
- Determine intervals of when a function is increasing/decreasing.
- Use the First Derivative test to determine relative extrema.
- Determine absolute extrema using the appropriate methods.
- Determine concavity of functions over their domain.
- Use the second derivative test to determine extrema.
- Sketch graphs of functions using their first and second derivatives.
- Solve optimalization problems.
- Explore behaviors of implicit relations.

Unit 6: Integration and Accumulation of Change

Description:

Unit 6 introduces the meaning of accumulations of change. Riemann Sums will be used to approximate areas and connect topics of summation notation and the definite integral notations. The Fundamental Theorem, both Part 1 and Part 2, will be discussed so that students can interpret the behavior of accumulations functions involving area. Indefinite and definite integrals will be taught along with techniques to integrate various functions. Anti-differentiation will be used to connect position, velocity and acceleration. Improper integrals are introduced and how to evaluate them.

Skills to be demonstrated:

- Explore accumulations of change.
- Approximate areas using Riemann Sums.
- Connect topics of Riemann Sums, summation notation and definite integral notation.
- Learn the Fundamental Theorem of Calculus and accumulation functions as well as with definite integrals.
- Interpret the behavior of accumulation functions involving area.
- Apply properties of definite integrals.
- Connect position, velocity and acceleration functions using integrals. Find total distance traveled.
- Using accumulation functions and definite integrals in applied contexts.
- Find antiderivatives and indefinite integrals (basic rules and notation).
- Integrate using substitution.
- Integrate functions using long division and completing the square. Evaluate integrals using Integration by Parts.
- Evaluate integrals using Partial Fractions.
- Discern which technique is needed for anti-differentiation.
- Find the arc length of a smooth planar curve defined on a closed interval.
- Discern the difference between proper and improper integrals.
- Understand why an integral may be improper.
- Evaluate improper integrals using limits.

Unit 7: Differential Equations

Description:

Unit 7 introduced differential equations so that situations can be modeled. Students will verify solutions using differential equations. Sketching slope fields will be explored and students will be able to make conclusions using slope fields. Students will use separation of variables to find general and particular solutions including exponential solutions. Logistic Growth models will be discussed.

Skills to be demonstrated:

- Model situations using differential equations.
- Verify solutions for differential equations.
- Sketch slope fields.
- Make conclusions using slope fields.
- Find general solutions and particular solutions using the integration technique of separation of variables.
- Use exponential models with differential equations. Use Euler's Method with a given step size to approximate values for the solution of an initial-value problem.
- Use slope fields to model logistic growth.
- Use logistic growth models to find the carrying capacity of populations.
- Use logistic growth models to discern when a population is increasing/decreasing and when the population's growth is the highest.
- Given an initial population of a logistic growth model, find an equation for the population after a given time period.

Unit 8: Applications of Integration

Description:

Unit 8 introduces average value of a function in various contexts. Students will find areas between curves expressed as functions of x or y. Finding volumes using cross sections, disk/washer method and shell methods will end this unit.

Skills to be demonstrated:

- Find average value of a function on an interval.
- Find the area between curves expressed as functions of x and y.
- Find the area between curves that intersect at more than two points.
- Find volumes using cross sections (squares, rectangles, semi circles, equilateral triangles).
- Find volumes using the disc/washer method revolving around the x, y axis and other axes.
- Find volumes using the shell method revolving around the x, y axis and other axes.

Unit 9: Sequences and Series

Description:

Unit 9 introduces sequences and series. Convergence and divergence will be explored along with using various tests to determine if a series is convergent or divergent. Error bounds will be found with alternating series. Absolute and conditional convergence will be discussed primarily with the root and ratio tests. Power Series, Taylor and Maclaurin series will be developed to represent functions. Their radius and intervals of convergence will be found. Lagrange Error bound will also be discussed.

Skills to be demonstrated:

- Understand the difference between a sequence and a series.
- Determine when a sequence is convergent or divergent.
- Find nth partial sum of a series.
- Define a geometric series and know when it converges or not.
- Use the nth term test for divergence of a series
- Use the integral test for show convergence/divergence of a series.
- Use the harmonic series and p-series to show convergence/divergence of a series.
- Use comparison tests to show convergence/divergence of a series.
- Use the alternating series test to show convergence of a series.
- Use the ratio and root test to show convergence/divergence of a series.
- Determine absolute or conditional convergence of a series.
- Find the error bound in an alternating series.
- Find Taylor Polynomial approximations of functions.
- Find the Lagrange Error bound.
- Find the radius and interval of convergence of power series.
- Find Taylor and Maclaurin series for a function.
- Represent functions as power series.

Unit 10: Parametric Curves and Vectors

Description:

Unit 10 introduces parametric equations and how to use them to solve calculus problems. Starting with defining and differentiating parametric equations, students then will find equations of tangent lines. Using a graphing tool, students will compare graphs to their calculus computations to confirm slopes of tangent lines including vertical and horizontal tangent lines. The second derivative will be taught and used to show concavity of the parametric curve. Arc Lengths of parametric curves will be explored and compared with arc lengths of smooth planar curves. Defining and differentiating and integrating vector-valued functions will be explored. Students will solve motion problems using parametric and vector-valued functions.

Skills to be demonstrated:

- Define and graph a parametric equation.
- Differentiate parametric equations, both first and second derivatives.
- Find the arc length of curves of parametric equations.
- Find equations of tangent lines including vertical and horizontal tangents.
- Compare written calculus computations for tangent lines to graphs.
- Find concavity intervals for parametric curves.
- Define vector-valued functions.
- Differentiate and integrate vector-valued functions.
- Solve motion problems using parametric and vector-valued functions.

Unit 11: Polar Curves

Description:

Unit 11 introduces polar coordinates and graphing using polar curves. The area of a polar region, the area bounded by a single polar curve, or the area of the region bounded by two polar curves will be discussed.

Skills to be demonstrated:

- Graph polar coordinates and graphs.
- Convert from polar coordinates to rectangular coordinates and vice versa.
- Differentiate a polar equation.
- Use differentiation, to find the equations of tangent lines to a polar curve.
- Find the area of a polar region.
- Find the area bounded by a single polar curve.
- Find the area of the region bounded by two polar curves.