HSML Honors Calculus II Course Preparedness Test

High School Math Live wants parents to be well informed. We want your student to be placed in the appropriate course so that they will be successful and challenged while learning the beauty of mathematics. Our year-long Honors Calculus II course is a college level course, equivalent to what most universities and colleges teach in one semester. Our pace is brisk and it is extremely important that your student have the needed skills to be successful in this rigorous course. The questions we have below will be a good indicator to whether your student is ready for our course. Completing approximately 80% correctly is a good sign that your student is ready for this level of Calculus. There is no time limitation for completing this but if your student takes longer than 2 hours, it might suggest that this course would require more time designated for math practice in their schedule. If your student struggles with the majority of the questions, then you and your student will need discuss your plans. Will your student simply need to allow more time for math this year? Will your student need a private tutor? Will your student put forth the effort needed in order to be successful? We cannot answer these questions for you, but we want you to know prior to the school year what skills are needed for this course.

The answer key is provided at the end of the document. We ask that you sit down with your student and discuss their work. It is very important that you are aware of their results so that informative decisions can be made. After completing these problems, if you find that there are concepts that your student has not mastered, by request, we are willing to add a workshop to our Brush Up on Math Workshops to get targeted help on those concepts. If you feel that input from an instructor of the course would be helpful, please scan their work and answers, making sure that the answers are in the same order as the problems on the document. The scan should be emailed as a single PDF document to support@highschoolmathlive.com.

Please view our website, www.highschoolmathlive.com, to read other details including what makes a successful HSML student and online learner.

ALL QUESTIONS SHOULD BE ANSWERED **WITHOUT** A CALCULATOR.

Note: Calculus students should be able to use their graphing calculator to:

1) *Locate the maximum/minimum points*
2) *Locate points of intersection*
3) *Locate the zeros of the function*
4) *Graph functions in an appropriate viewing window*

Also, Calculus students need to be able to answer questions regarding the Unit Circle without a calculator.

Calculus I topics that are prerequisites for Honors Calculus II:

- *Limits including L'Hospital's Rule and Limits at Infinity*
- *Continuity*
- *Derivatives and Rates of Change*
- *Differentiation Rules including Product and Quotient Rules, Chain Rule*
- *Implicit Differentiation*
- *Related Rates and Optimization Problems*
- *Straight-Line Motion: position, velocity, acceleration*
- *Applications of the derivative including graphing using Calculus techniques*
- *Riemann Sums, Integration, Definite/Indefinite Integrals, Substitution Rule*
- *Areas/Volumes*
- *Differential Equations, Direction (Slope) Fields and Separable Equations*

Part 1: Find the values of the following.

1. $\sin\left(\frac{\pi}{3}\right)$
2. $\cos(\frac{3\pi}{2})$
3. $\tan(\frac{3\pi}{4})$
4. $\csc(\frac{7\pi}{6})$

Part 2: Evaluate the following limits if it exists. State *Does Not Exist* if applicable.

1. $\lim\limits_{x\to 1}\dfrac{x^2-1}{x^2+2x-3}$
2. Let $f(x)=\begin{cases} 5x-x^2 & \text{if } x<1 \\ 2-x & \text{if } x\geq 1 \end{cases}$ find $\lim\limits_{x\to 1^-} f(x)$
3. $\lim\limits_{x\to 5}\dfrac{e^x}{(x-5)^2}$
4. $\lim\limits_{x\to 4}\dfrac{\frac{1}{4}-\frac{1}{x}}{4-x}$
5. $\lim\limits_{x\to\infty}\dfrac{x^3}{e^{x^2}}$

Part 3: State where f(x) is continuous. State answers using interval notation.

1. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$
2. $f(x) = \begin{cases} 1 + x^2 & if \quad x \le 0 \\ 2 - x & if \quad 0 < x \le 2 \\ (x-2)^2 & if \quad x > 2 \end{cases}$

Part 4: Find the values of a and b that makes f(x) continuous and differentiable for all real numbers.

$$f(x) = \begin{cases} x^2 & if \quad x \le 2 \\ ax + b & if \quad x > 2 \end{cases}$$

Part 5: Differentiate each function. Simplify all except #1.

1. $f(x) = \left(x^2 - 5x + 2\right)^4$
2. $f(x) = \dfrac{5x}{\left(x^2 + 3\right)^2}$
3. $f(x) = e^{-5x^2} \sin(4x)$
4. $f(x) = \ln(x^2 - 4)$

Part 6: Various topics on differentiation.

1. Find the equation of the tangent line (in point slope form) to the curve $f(x) = \sqrt{1 + x^3}$ at the point (2, 3).

2. Find the x-coordinate(s) of the horizontal tangent of the curve: $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 1$

3. If $h(x) = f(g(x))$, where $f(-2) = 8,\ f'(-2) = -4,\ f'(5) = 3,\ g(5) = -2,$ and $g'(5) = 4$ find $h'(5)$.

4. Using implicit differentiation, find $\dfrac{dy}{dx}$. $\qquad x^2 y - 6x^2 = y^3 - 4$

5. Find the absolute maximum and minimum values of $f(x) = x\sqrt{4 - x^2}$ on $[-1, 2]$.

Part 7: For $f(x)=3x^5+5x^4$ state:

a. The x-coordinate of the critical points
b. Increasing/decreasing intervals (use interval notation)
c. The x-coordinates of local (relative) maximum, minimum points
d. Concave up/concave down intervals (use interval notation)
e. The x-coordinates of the point(s) of inflection

Part 8: Antiderivatives

1. Find f(x) given: $f''(x)=4+6x+24x^2,\ f(1)=-1,\ f'(-1)=3$
2. Given: a(t) = acceleration function, v(t) = velocity function and s(t) = position function.
   If $a(t)=2t+1,\ v(0)=3,\ s(0)=2$ find the position function s(t).

Part 9: Integration. Find either the general indefinite integral or evaluate the integral.

1. $\int_{-1}^{1}(x^3+\sqrt[3]{x}-2)dx$
2. $\int e^{-8x}dx$
3. $\int (3-x^3)^3 x^2 dx$
4. $\int x\sin(3x^2+5)dx$
5. $\int_e^2 \frac{1}{x\ln x}dx$
6. $\int \frac{1}{x^2+9}dx$
7. $\int \frac{x+1}{x^2+2x+5}dx$
8. $\int_0^{\pi/3}(5\sin x-\cos x)dx$
9. $\int \frac{e^{3x}}{\sqrt{1-e^{3x}}}dx$
10. $\int \frac{e^x}{1+3e^x}dx$

Part 10: Areas/Volumes

1. Set up, do not evaluate, the integral to find the area of the region enclosed by the curves:
   a. $y = 12 - x^2$ and $y = x^2 - 6$
   b. $x = 1 - y^2$ and $x = y^2 - 1$

2. Set up, do not evaluate the integral to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Use Disk/Washer methods.
   a. $y = \sqrt{x-1},\ y = 0,\ x = 5$ about the x-axis
   b. $y^2 = x,\ x = 2y$ about the y–axis.

Part 11: Find the solution of the differential equation that satisfies the given initial condition.

1. $\dfrac{dy}{dx} = xe^{-y},\ \ y(0) = 0$
2. $2yx\dfrac{dy}{dx} = (x+1),\ \ y(e) = 1$

Solutions:

Part 1: 1. $\frac{\sqrt{3}}{2}$ 2. 0 3. -1 4. -2

Part 2: 1. ½ 2. 4 3. ∞ or Does Not Exist 4. $\frac{-1}{16}$ 5. 0

Part 3: 1. $(-\infty,2)\cup(2,\infty)$ 2. $(-\infty,0]\cup(0,\infty)$

Part 4: a = 4 and b = -4

Part 5: 1. $f'(x) = 4(x^2-5x+2)^3(2x-5)$ 2. $f'(x) = \dfrac{-15x^2+15}{(x^2+3)^3}$

3. $f'(x) = 2e^{-5x^2}(2\cos 4x\ \text{-}\ 5x\sin 4x)$ 4. $f'(x) = \dfrac{2x}{x^2-4}$

Part 6: 1. $y-3 = 2(x-2)$ 2. X = -2, 3 3. -16 4. $\dfrac{dy}{dx} = \dfrac{-2xy+12x}{x^2-3y^2}$

5. absolute max value is 2 found at $x = \sqrt{2}$ and the absolute min value is $-\sqrt{3}$ found at x = -1

Part 7: a. x = 0 and -4/3

b. f is increasing on $(-\infty, {}^{-4}\!/_3) \cup (0,\infty)$ and f is decreasing on $({}^{-4}\!/_3, 0)$

c. x = -4/3 gives local max and x = 0 gives local min

d. f is concave up on $(-1,0) \cup (0,\infty)$ and f is concave down on $(-\infty,-1)$

e. f has a point of inflection at x = -1

Part 8: 1. $f(x) = 2x^2 + x^3 + 2x^4 + 12x - 18$ 2. $s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 + 3t + 2$

Part 9: 1. -4 2. $\frac{-1}{8}e^{-8x} + C$ 3. $\frac{-1}{12}\left(3 - x^3\right)^4 + C$ 4. $\frac{-1}{6}\cos(3x^2 + 5) + C$

5. $\ln(\ln 2)$ 6. $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$ 7. $\frac{1}{2}\ln\left|x^2 + 2x + 5\right| + C$

8. $\frac{5}{2} - \frac{\sqrt{3}}{2}$ 9. $\frac{-2}{3}\left(1 - e^{3x}\right)^{1/2} + C$ 10. $\frac{1}{3}\ln\left|1 + 3e^x\right| + C$

Part 10: 1a. $\int_{-3}^{3}\left((12 - x^2) - (x^2 - 6)\right)dx$ 1b. $\int_{-1}^{1}\left((1 - y^2) - (y^2 - 1)\right)dy$

2a. $\int_{1}^{5}\pi\left(\sqrt{x-1}\right)^2 dx$ 2b. $\int_{0}^{2}\left(\pi(2y)^2 - \pi\left(y^2\right)^2\right)dy$

Part 11: 1. $y = \ln\left(\frac{1}{2}x^2 + 1\right)$ 2. $y = \sqrt{x + \ln|x| - e}$