

## Algebra IIA/IIB

### Course Overview:

This course is designed to review and extend the topics from Algebra I and Geometry while providing a solid foundation in preparation for Pre-Calculus. Students relate the arithmetic of rational expressions to the arithmetic of rational numbers, expand functions to include polynomial, rational, radical, logarithmic, exponential, and trigonometric functions, and explore various data collection methods to relate data and statistics to probability. This course includes a strong emphasis on developing critical thinking and analytical reasoning skills through the study of real-world applications throughout each topic.

### Assignments and Assessments:

Guided Learning Units - Each semester, students will be assigned a Guided Learning Unit that is divided into 4 parts. Each part will be due on specific dates throughout the semester and will be worth 25 points. The project will be worth a total of 100 point and will count as one test grade.

All other units will be completed following this process:

1. Students will attend live and interactive classes and participate by answering questions via the private chat with the teacher. By doing so, students will have immediate feedback on their current level of comprehension, thus reinforcing learning and providing the opportunity to improve their skills within each lesson.
2. Students will complete one homework assignment before attending the next instruction session. Students are provided with answers to homework assignments and are expected to self-check their answers. Classes begin with time for students to ask the instructor about homework questions they were not able to complete correctly.
3. Once each week, students complete a quiz (using paper and pencil) that is proctored by an adult and returned to the instructor for grading and feedback. Instructors look at both work and answers to ensure that students are using sound mathematical processes to demonstrate mastery. After each unit, students complete a test in the same manner. The instructor creates an online personal grading notebook for each student where the student's work, the instructor's feedback, and the grade can be viewed. Access to the notebook is granted to the student and parents, as well as any education advisors.
4. At the end of each semester, students complete a cumulative final exam.
5. Course grades are assigned based on a weighted average of 40% quiz / 60% test. The final exam is weighted the same as the other tests.

## **Unit 1: Data Analysis [Guided Learning Unit – fall semester]**

**Description:** Unit 1 will focus on statistical principles such as normal distribution using the mean and standard deviation to make inferences and approximate population percentages while recognizing that some data sets do not fit a normal distribution curve. Students will understand and evaluate random processes using statistical experiments so that inferences can be made regarding a population set. Using sample surveys, experiments and other observational studies, students will make inferences and justify any conclusions that can be drawn, recognizing when conclusions cannot be drawn. Finally, students will use probability to evaluate outcomes of decisions, determine fairness and analyze decisions and strategies.

### **Skills to be demonstrated:**

- Use the mean and standard deviation of a set of data (sample or population) to fit it to a normal distribution.
- Find the areas and population percentages under a normal distribution using z-scores or calculator-based apps.
- Discern when a set of data models a normal distribution.
- Analyze data to learn when inferences regarding population can be made.
- Recognize possible associations and trends from random samples, simulations, surveys, observational studies, and experiments.
- Learn the purposes of and the differences among random samples, simulations, surveys, observational studies, and experiments and explain how randomization relates to each.
- Using data gathered from a sample survey, estimate a population mean or proportion.
- Develop the margin of error using simulation models for random sampling.
- Use data from a randomized experiment to compare two treatments.
- Use simulations to decide if differences between parameters are significant.
- Evaluate reports given various forms of data and decide if a sample is biased or misleading.
- Use probabilities to make fair decisions.
- Using probability concepts, analyze decisions and strategies.

## **Unit 2: Trigonometric Functions [Guided Learning Unit – spring semester]**

**Description:** Unit 2 will focus on the Unit Circle using radian measurements. Evaluating six trigonometric functions using radian measurement traversed counterclockwise around the Unit Circle. The graphs of all six basic trigonometric functions will be explored while modeling periodic phenomena with appropriate trigonometric functions. Amplitude, frequency and midline will be discussed. Trigonometric identities and a given quadrant of the angle will be used to evaluate the six trigonometric functions.

### **Skills to be demonstrated:**

- Understand radian measurement as the length of the arc on the unit circle subtended by the angle.
- Evaluate the basic six trigonometric functions using radian measurement and the Unit Circle.
- Graph the basic six trigonometric functions noting the amplitude, frequency and midline.
- Model periodic phenomenon using amplitude, frequency and the midline.
- Understand the Pythagorean Identity.
- Use the Pythagorean Identity and a given quadrant of the angle to evaluate the six trigonometric functions.

### Unit 3: Linear Equations and Functions

**Description:** Unit 3 focuses on exploring linear equations and inequalities with between one and three variables, an introduction to function notations and terminology, and matrices. Students will discover the relationship between algebraic solutions, solutions from a matrix using multiple strategies, and graphical representations of functions and will utilize given information to write and solve equations.

#### Skills to be demonstrated:

- Simplify and solve multi-step equations with one variable.
- Simplify and solve one variable simple and compound inequalities using intersection and union of sets, giving answers in graph and interval forms and in set notation.
- Simplify and solve absolute value equations and inequalities using intersection and union of sets, giving answers in graph and interval form and in set notation.
- Solve linear equations for a given variable to rearrange the formula to highlight a quantity of interest or identify equivalent expressions.
- Write one- and two-variable equations based on given information and solve real world problems including mixtures, investments, uniform motion, and angles.
- Solve systems of two-variable linear equations and inequalities algebraically and graphically.
- Use linear systems to write equations of piecewise functions and discuss continuity.
- Explain the relationship between intersections and solutions in systems of linear functions.
- Solve a one-variable linear equation by graphing a related system of equations.
- Solve systems of linear equations involving three equations and three variables.
- Write equations based on given information to solve real world problems including two equations with two variables and three equations with three variables.
- Distinguish between dependent and independent variables, relations and functions.
- Find the domain and range of linear functions, constant functions, and vertical lines.
- Find the domain and range of non-linear graphs.
- Find domain of non-linear functions.
- Given a function that describes a real world situation, use the function to find requested information.
- Solve systems of  $3 \times 3$  linear equations using matrices, without using a calculator.
- Find the determinant of a  $2 \times 2$  matrix.
- Solve  $2 \times 2$  matrices using Cramer's Rule.

#### Unit 4: Polynomial Functions and Equations

**Description:** Unit 4 focuses on exploring the structural similarities between the system of integers and the system of polynomials. Students will identify the zeros of polynomials using both function notation and the Remainder Theorem. Students will make connections between the zeros of the polynomials and the solutions of polynomial equations. Students will apply Synthetic Division and the Remainder Theorem to discover the behavior of a function.

##### Skills to be demonstrated:

- Add, subtract, multiply and divide polynomials.
- Rewrite expressions of  $\frac{a(x)}{b(x)}$  in the form of  $q(x) + \frac{r(x)}{b(x)}$  where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ .
- Know the Remainder Theorem and apply to determine roots and evaluate the behavior of a function.
- Use Synthetic Division to determine whether a given number is a solution of an equation.
- Solve composition of function problems.
- Write equations using composition of functions to interpret real world data.
- Use factoring to solve equations that are in quadratic form.
- Use factoring to solve equations of perfect cubes.
- Use factoring by grouping to solve equations that have a degree higher than 2.
- Model and solve real world problems involving polynomials, for example physics and volume applications.

#### Unit 5: Rational Expressions and Functions

**Description:** Unit 5 focuses on relating the arithmetic of rational numbers to the arithmetic of rational expressions. Students will develop abstract reasoning skills to simplify and solve rational expressions and functions as well as real world applications of increasing complexity.

##### Skills to be demonstrated:

- Add, subtract, multiply and divide simple rational expressions, recognizing that rational expressions are governed by the same rules as the arithmetic of rational numbers.
- Simplify complex rational expressions.
- Solve simple rational equations and determine extraneous solutions.
- Graph rational functions by recognizing key features including domain, range, and asymptotes.
- Discuss rational function graphs in terms of increasing or decreasing, relative max and mins, symmetry, and end behavior.
- Write rational equations to solve real word problems, including interpreting the solution in context and determining extraneous roots, (eg proportions,  $d=rt$ , and work rates.)
- Solve rational formulas for specified variables.

## Unit 6: Radical and Root Functions

**Description:** Unit 6 focuses on extending their previous learning of simplifying radicals and working with rational exponents to problems with a higher level of complexity including complex zeros and using conjugates. Students will improve skills in computation with radicals and complex numbers.

### Skills to be demonstrated:

- Add, subtract, multiply, and divide radical expressions.
- Apply the exponent rules and order of operations to simplify expressions with rational exponents which require multiple steps to simplify.
- Rationalize monomial and binomial denominators with various degrees.
- Multiply and divide radicals with different indices.
- Solve radical equations with indices greater than 2.
- Solve radical equations with multiple radicals and non-radical terms.
- Solve radical equations by rewriting in quadratic form eliminating extraneous solutions.
- Solve radical equations for a given variable.
- Apply transformations to parent functions to graph radical functions with indices of 2 and 3, identifying domain and range.
- Add, subtract, multiply, and divide complex numbers.
- Find powers of  $i$ .
- Apply the Complex Conjugate Theorem to rationalize complex denominators.

## Unit 7: Quadratic Equations, Inequalities, and Functions

**Description:** Unit 7 focuses on solving quadratic equations and inequalities using a variety of skills, applying the relationships between factors, zeros, and intercepts in real world problem solving, and transforming graphs. Students will identify key features of graphs that will result in a comprehensive understanding of how to interpret real world problems using equations and graphs.

### Skills to be demonstrated:

- Solve quadratic equations with real or complex roots by using the square root property, completing the square, the quadratic formula, and by graphing.
- Derive the quadratic formula by completing the square.
- Rewrite quadratics with complex solutions in root form.
- Use factoring to solve equations that are in quadratic form, including equations with degrees greater than 2 as well as equations with rational exponents.
- Solve equations that are quadratic but do not have a degree of two.
- Solve rational equations that simplify to become quadratic equations, identifying extraneous solutions.
- Write quadratic equations and solve real word applications including Pythagorean Theorem, area, work rates, and quadratic models.
- Solve formulas for specific variables including squares and square roots
- Graph quadratic functions using transformations.
- Given an equation, interpret the key features and sketch the graphs of quadratic functions using intercepts, intervals where the function is increasing / decreasing, positive/negative values, relative maximums / minimums, axis of symmetry, and end behavior.
- Calculate and interpret the average rate of change of a quadratic function over a specified interval.
- Explain the relationship between intersections and solutions in systems of functions of all types.
- Use the discriminant to determine the qualities of a quadratic function.
- Graph parabolas with horizontal axis of symmetry.
- Solve real world problems involving maximum and minimum values of quadratic equations, including eliminating extraneous solutions.
- Solve inequalities including quadratic, polynomial with degree 3 or greater, and rational.

## Unit 8: Inverse, Exponential, and Logarithmic Functions

**Description:** Unit 8 focuses on introducing the concepts of inverse, exponential, and logarithmic functions, along with the value of  $e$  and natural logarithms. Students will recognize the relationships between functions and their inverses and learn to solve and graph both exponential and logarithmic equations.

### Skills to be demonstrated:

- Determine whether a function is one-to-one using algebraic and graphing criteria.
- Find the equation of the inverse function given an equation of a one-to-one function, explain the relationships between the domains and ranges of a function and its inverse.
- Graph the inverse of a one-to-one linear and non-linear function given a graph of the function.
- Determine the inverse of a quadratic function and define the domain and range for which the inverse exists.
- Solve exponential equations.
- Solve real world applications of exponential functions such as growth and decay.
- Convert between logarithmic and exponential forms and find the exact values of logarithms.
- Identify exponential and logarithmic functions as inverses and explain the relationship between their graphs.
- Prove simple laws of logarithms.
- Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.
- Solve logarithmic functions that model real world data to find results for given values.
- Graph exponential and logarithmic functions, including those with transformations to the parent function, show their intercepts and end behaviors, identify the domain and range.
- Know the number  $e$  and its purpose in exponential and logarithmic functions.
- Solve exponential and logarithmic equations, including base 10, natural logs, and other bases.
- Solve exponential functions that model real world data to find results for given values.

## Unit 9: Nonlinear Functions and Systems, Circles, Parabolas, Ellipse and Hyperbola

**Description:** Unit 9 focuses on a holistic overview of the properties of nonlinear functions and their graphs, solving nonlinear systems of equations and inequalities, and deepening the understanding of circles and parabolas both graphically and algebraically. Students will be introduced to equations of ellipse and hyperbola and will explore the effects of transformations on diverse functions, including functions arising in an application.

### Skills to demonstrate:

- For absolute value, rational, and radical functions, identify key features including domain and range, intercepts, end behavior, any existing vertical or horizontal asymptotes, and any holes in the graph.
- For absolute value, rational, and radical functions, graph by applying transformations to parent functions.
- For absolute value, rational, and radical functions, sketch the graph by identifying intervals where a given function is increasing or decreasing, relative maximums and minimums, and symmetries.
- For absolute value, rational, and radical functions, recognize and use parent functions to identify the equation of a function that has been transformed.
- Solve real world problems involving step functions and graph the step function.
- Compare and observe properties of two functions represented in algebraic, graphical, numerical forms, or with verbal descriptions.
- Know and apply the Binomial Theorem for the expansion of the  $(x+y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.
- Given a quadratic equation, use the method of completing the square to put the equation into standard form for parabolas and circles.
- Given equations in standard form, graph ellipses and hyperbola including those with translated centers.
- Given equations not in standard form, use basic algebra skills to write the equation of an ellipse and a hyperbola in standard form.
- Given an equation in any form, graph parabola, circle, ellipse and hyperbola.
- Demonstrate the parallel methods of solving linear and second degree systems of equations using algebra.
- Demonstrate the parallel methods of solving linear and second degree systems of inequalities by graphing.