

Honors Pre-Calculus A/B

Course Description:

This course provides students with the background they need to succeed in calculus. It emphasizes the demonstration of strong algebra skills in increasingly complex problems, applying skills to real world applications, and using technology to enhance comprehension of abstract concepts.

Assignments and Assessments:

Each unit will be completed following this process:

1. Students will attend live and interactive classes or view recorded (At My Pace - AMP) instruction video and participate by answering questions in the live setting or by pausing the AMP instruction video to solve a given problem. Students will have immediate feedback on their current level of comprehension, thus reinforcing learning and providing the opportunity to improve their skills within each lesson.
2. Students will complete one homework assignment before attending or viewing the next instruction session. Students are provided with answers to homework assignments and are expected to self-check their answers. Classes begin with time for students to ask the instructor about homework questions they were not able to complete correctly. Students using AMP recorded instruction can set up time with an instructor, up to 5 hours per course, for individual tutoring.
3. After approximately every two assignments, students complete a quiz (using paper and pencil) that is proctored by an adult and returned to the instructor for grading and feedback. Instructors look at both work and answers to ensure that students are using sound mathematical processes to demonstrate mastery. Students complete a tests over larger sections of material in the same manner. The instructor creates an online personal grading notebook for each student where the student's work, the instructor's feedback, and the grade can be viewed. Access to the notebook is granted to the student and parents, as well as any education advisors.
4. At the end of each semester, students complete a cumulative final exam.
5. Course grades are assigned based on a weighted average of 40% quiz / 60% test. The final exam is weighted the same as the other tests.

Unit 1: Functions, Systems, and Matrices

Description:

Unit 1 focuses on deepening the student's knowledge of functions and the use of appropriate terminology, computation and interpretation of matrices, and applications of functions involving linear programming.

Skills to be demonstrated:

- Given the equation of a function, graph and describe the function using terminology such as one-to-one, domain, range, increasing, decreasing, end behavior, continuity, even/odd/neither.
- Given a piece-wise function, evaluate the function for given values and graph the function.
- Graph transformation of families of functions that have been translated, reflected, and rotated.
- Write equations of a function given a graph of a family of functions that has been transformed.
- Given a composition of functions, evaluate the function for a given value.
- Given two equations, write a composition of functions.
- Solve systems of equations using substitution and elimination, graphing, and technology.
- Solve systems of inequalities.
- Use matrices to represent sets of data and systems of equations.
- Solve systems of three equations using matrices with and without technology.
- Find the determinant of a three by three matrix with and without technology.
- Solve matrices using Cramer's Rule.
- Find the partial fraction decomposition for rational expressions and using matrices to solve for variables when necessary.
- Solve systems of equations using inverse matrices with and without technology.
- Multiply a matrix by a scalar and by another matrix.
- Show that matrix multiplication is not always possible and is not commutative.
- Solve applied problems using matrices.
- Model and solve applied problems using linear programming.

Unit 2: Quadratics

Description:

Unit 2 focuses on deepening the understanding of quadratic functions including terminology and application.

Skills to be demonstrated:

- Solve quadratic equations algebraically using multiple methods including factoring, quadratic formula, completing the square, and graphing.
- Solve quadratic inequalities, stating answer in interval form.
- Interpret the key features, sketch the graph of, and give the equation of a quadratic functions using intercepts, intervals where the function is increasing/decreasing, positive/negative values, relative maximums/minimums, axis of symmetry, and end behavior.
- Use the discriminant to describe the roots of a function.
- Model and solve real world problems that are described by a quadratic function.
- Solve and interpret applied problems, such as physics.

Unit 3: Polynomials and Rational Functions

Description:

Unit 3 focuses on deepening the student's understanding of polynomials and rational functions. Continuing to apply previously learned skills, students encounter more complex polynomials and find the data necessary to graph and interpret the given equations. Students also use graphs to determine properties of equations and write equations that match given graphs.

Skills to be demonstrated:

- Factor polynomial expressions using the Fundamental Theorem of Algebra and the Remainder Theorem.
- Rewrite expressions of $\frac{a(x)}{b(x)}$ in the form of $q(x) + \frac{r(x)}{b(x)}$ where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ using both long division and synthetic division.
- Find zeros of polynomials with degrees greater than 2 and graph the polynomial.
- Identify a polynomial equation with a degree greater than 2 that has matching characteristics to a given a graph of the function using Fundamental Theorem of Algebra, multiplicity, end behavior, positive and negative values of the function, and intercepts.
- Find complex roots of a polynomial equation, applying the Complex Conjugate Theorem.
- Use Descartes' Rule of Signs to determine the number of positive and negative zeros of a polynomial function.
- Simplify rational expressions to determine the vertical, horizontal, and oblique asymptotes of rational functions, holes in the function, and their domain and range.
- Use the key features of a rational function to sketch its graph.
- Solve applied problems involving rational function equations.
- Solve applied problems using direct, inverse, and joint variation.

Unit 4: Inverse, Logarithmic, and Exponential Functions

Description:

Unit 4 focuses on the topics of logarithmic and exponential functions, combining the inverse relationship of those functions with the idea of inverses in all functions. Real world applications are a significant focus and students solve increasingly complex logarithmic and exponential equations.

Skills to be demonstrated:

- Solve for the inverse of a function, give the domain and range of both restricting the domain if necessary to make the functions one-to-one.
- Determine whether functions are inverses by using composite functions.
- Solve exponential and logarithmic equations.
- Solve applied problems using compound interest including continuous compounding.
- Convert between logarithmic and exponential forms to find the exact or estimate of the values of logarithms.
- Use the properties of logarithms to expand and simplify logarithmic expressions.
- Graph exponential and logarithmic functions, including those with transformations to the parent function, stating the domain, range, intercepts, asymptotes, and whether the function is increasing or decreasing.
- Solve common and natural logarithmic equations using the change of base formula.
- Solve applied problems of exponential problems, for example growth / decay, and for functions with logarithmic terms.

Unit 5: Trigonometric Functions

Description:

Unit 5 focuses on reviewing and increasing comprehension of Trigonometric functions and how they relate to real word applications.

Skills to be demonstrated:

- Identify the six trigonometric values of angle measures using the unit circle.
- Convert angles between radian and degree measures, and vice versa.
- Find the arclength and area of a sector given a central angle and radius.
- Solve real world problems involving angular speed and linear velocity.
- Graph the six trigonometric functions and restrict the domains of their inverses so that their graphs are also functions.
- Determine the amplitude and period, identify properties, and graph the sine and cosine functions.
- Model and solve real world problems involving trigonometric equations in which their solution is found using inverse trig functions.
- Solve applied problems, including those with bearing terminology, that result in trigonometric equations.
- Solve applied problems using simple harmonic motion.

Unit 6: Trigonometric Identities

Description:

Unit 6 focuses on the logic of deriving trigonometric identities and solving trigonometric equations. Real world applications are demonstrated as an application of the Law of Sines and Law of Cosines.

Skills to be demonstrated:

- Verify and derive trigonometric identities using other identities.
- Solve trigonometric equations.
- Derive the Law of Sines and the Law of Cosines.
- Solve applied problems using the Law of Sines and the Law of Cosines.
- Derive the area formula for triangles using the Law of Sines.
- Solve equations involving inverse trigonometric functions.

Unit 7: Vectors

Description:

Unit 7 focuses on learning to apply the concepts of trigonometry to vectors and the real world applications that come from the study of vectors.

Skills to be demonstrated:

- Recognize vectors quantities have a magnitude and direction, and calculate a vector's magnitude.
- Given the magnitude and direction of a vector, find the vector's horizontal and vertical components.
- Use appropriate symbols for vectors and their magnitudes.
- Use the additive inverse property to solve vector subtraction problems.
- Add or subtract vectors end-to-end, component-wise, and by the parallelogram rule (for addition only.)
- Multiply and transform vectors using the dot product and cross product.
- Determine whether two vectors are orthogonal.
- Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- Represent scalar multiplications graphically and perform scalar multiplication component-wise.
- Calculate a vector sum's magnitude and direction.

- Find the magnitude and direction of an equilibrant vector.
- Model and solve real world problems using vectors and trigonometric ratios including force and navigation problems.
- Convert equations between rectangular and trigonometric form.
- Use the product and quotient theorems to multiply and divide complex numbers in trigonometric form.

Unit 8 Non-Cartesian Representatives

Description:

Unit 8 focuses on graphs of complex numbers in rectangular form and polar form. Parametric equations are introduced and the application of parametric equations to real world problem solving is explored.

Skills to be demonstrated:

- Represent complex numbers on the complex plane in rectangular and polar form, convert between the two forms, and explain why they represent the same number.
- Find the modulus and argument of a complex number.
- Add, subtract, multiply, and divide complex numbers algebraically and graphically.
- Plot polar coordinates onto a polar graph, name other polar coordinates that represent the same point.
- Use technology to find points and then graph polar equations in sine and cosine.
- Use technology to graph polar equations.
- Represent parametric equations using a chart and a graph.
- Convert parametric equations to rectangular equations.
- Solve real world applications using parametric equations.

Unit 9: Conic Sections: Circles, Parabolas, Ellipses, and Hyperbolas

Description:

Unit 9 introduces all of the conic sections. Students transform equations to standard form in order to graph and discuss the various aspects of each conic section as well as write equations of conic sections given basic information about a graph. Real world applications using conic section equations are solved.

Skills to be demonstrated:

- Describe how to cut through a double cone to get each conic section.
- Identify the different parts of the standard equations for each conic section.
- Write the equations of each conic section given enough characteristic and values.
- Given a quadratic equation, complete the square to put the equation into standard form and identify which conic section it represents.
- Graph each conic section identifying applicable values including any of the following: domain, range, center, major and minor endpoints, vertices, foci, eccentricity, directrix, and asymptotes.
- Solve real world applications involving equations that represent conic sections.

Unit 10: Further Topics in Algebra

Description:

Unit 10 focuses on deeper applications on sequences that are arithmetic, geometric, and neither. Summation notation is introduced and related to the sum of a series. Finite and infinite sums are discussed.

Skills to be demonstrated:

- Identify arithmetic and geometric sequences.
- Given an explicit formula written as a polynomial or rational expression, find the n^{th} term of arithmetic and geometric sequences.
- Given a series written in summation notation, find the indicated sum both algebraically and with technology.
- Discuss the Fibonacci sequence as an example of a recursive sequence.
- Expand a polynomial using the binomial theorem.
- Distinguish between permutations and combinations, using both to solve problems involving counting theory.
- Review and expand the concepts of probability and odds including the probabilities of unions.